Electromagnetism

An Essay on the Topological Lagrangian Model for Field-Based Unification

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December 25, 2025

I Introduction

The unification of physics requires the derivation of Electromagnetism (EM) as a geometric necessity of the spacetime manifold. Standard physics identifies EM as a U(1) gauge theory, an internal symmetry group functioning independently of spacetime geometry [1]. The Topological Lagrangian Model (TLM) reinterprets this phenomenon: Electromagnetism constitutes the mechanical shear stress of the vacuum lattice. The field lines described by Faraday exist as physical twist lines in the non-orientable bulk.

This paper analyzes the electromagnetic sector of the TLM. We derive the photon, charge, and the fine structure constant from the topology of the Hyperbottle manifold [2]. Crucially, we demonstrate that Maxwell's equations arise inevitably as the linear limit of the topological shear, establishing the theory as a candidate for unification.

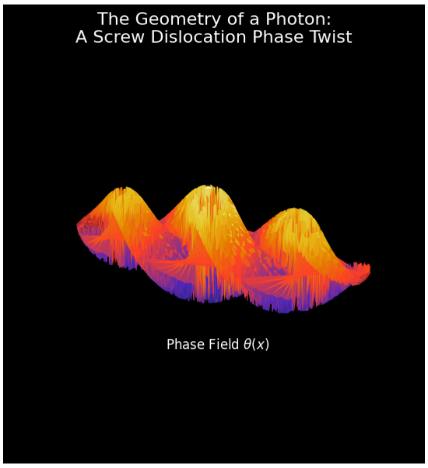

FIG. 1: **Screw Dislocation** A 3D helicoid surface representing the photon's phase structure twisting through the manifold.

II The Geometric Origin of the Photon

Standard models describe the photon as a point particle excitation of an abstract field. The TLM identifies the photon as a transverse shear wave propagating through the Phase-Locked lattice of the vacuum.

A The Screw Dislocation

We define the photon geometrically as a screw dislocation in the phase field $\theta(x)$. In crystallography, a screw dislocation represents a topological defect where the lattice planes form a helical ramp around the dislocation line. Analogously, the photon represents a propagating phase defect where the Intrinsic Vector field I^μ twists helically as it traverses the vacuum manifold [3]. This structure allows the wave packet to maintain coherence through the torsional stiffness of the manifold. Unlike the massive W/Z bosons which possess intrinsic topological mass due to choked geometry, the 'Heavy Photon' acquires effective mass only in high-shear regimes. This is analogous to a photon acquiring effective mass in a plasma or superconductor[1]; it is the drag of the vacuum lattice resisting the screw dislocation, not a change in the fundamental topology of the particle.

B Shear Wave Velocity

The speed of light c is determined by the elastic properties of the vacuum substrate. Within the TLM, c represents the Shear Wave Velocity of the manifold. This velocity is defined by the ratio of the vacuum's shear modulus γ to its coherence density β:

$$c = \sqrt{\frac{\gamma}{\beta}} \tag{1}$$

The shear modulus γ corresponds to the stiffness of the Planck-scale lattice (anchored to the Planck Area l_P^2), representing the resistance of the geometry to topological twisting. The coherence density β corresponds to the inertia of the Coherence Field, representing the pressure required to displace the field from its mean value. This relation unifies the causal limit of relativity with the mechanical properties of the topological substrate, consistent with the stability radius derivations found in the hadronic sector.

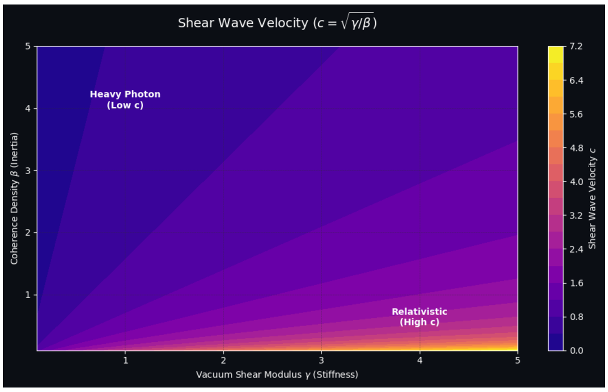

FIG. 2: **Shear Wave Velocity** A heatmap showing how the speed of light c emerges from the ratio of Shear Modulus (γ) to Coherence Density (β).

III Charge as Topological Winding

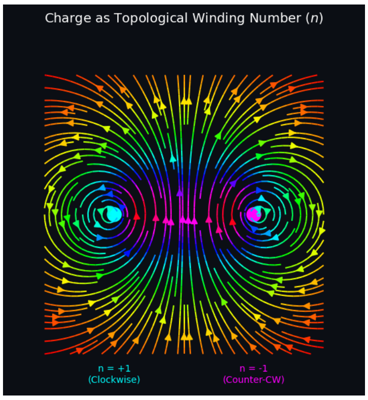

FIG. 3: **Winding Number N** A vector field streamplot illustrating the integer winding number $n = 1$ (Positive Charge) vs $n = -1$ (Negative Charge).

The TLM derives electric charge as a topological invariant. Standard theory treats charge as an intrinsic quantum number. We define it as the Winding Number of the Intrinsic Vector field I^μ around a topological defect [4].

$$Q = \oint_{\partial \mathcal{V}} I^\mu d\Sigma_\mu = 2\pi n \tag{2}$$

The Intrinsic Vector circulates around the core of a particle (soliton). The non-orientable manifold geometry compels the field to wind an integer number of times to remain single-valued. This quantization constraint explains the discrete nature of electric charge. Positive and negative charges correspond to clockwise ($n = +1$) and counter-clockwise ($n = -1$) twists relative to the manifold's orientation vector, matching the Helicity Invariant H derived in the particle stability analysis.

IV Resolution of the Abraham-Minkowski Controversy

The debate regarding photon momentum in a dielectric finds a resolution within this geometry. The controversy centers on whether photon momentum increases ($p = nE/c$) or decreases ($p = E/nc$) inside a medium. Experimental evidence supports both formulations depending on the measurement context [5].

The TLM separates the total momentum into two geometric sectors.

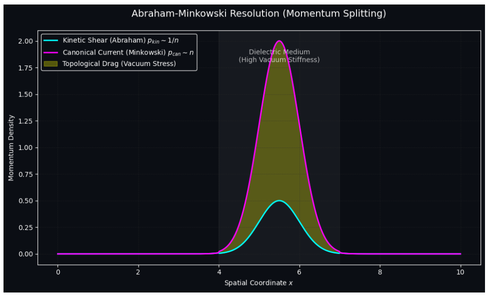

FIG. 4: **Abraham-Minkowski** A spatial plot showing the splitting of photon momentum into Kinetic (Abraham) and Canonical (Minkowski) components as it enters a stiff vacuum region.

1. *Kinetic Shear (Abraham Momentum)*

The Kinetic Shear vector p_{kin} represents the momentum of the wave packet relative to the lattice. This component describes the transport of the perturbation through the manifold, quantifying the energy flow as-

sociated purely with the displacement of the field am-
plitude. It treats the background geometry as a fixed stage, corresponding to the Abraham formulation where the momentum depends inversely on the refractive index ($p \propto 1/n$). This momentum sector governs the kinematic trajectory of the photon.

2. Canonical Current (Minkowski Momentum)

The Canonical Current vector p_{can} represents the total momentum of the system, including the Topological Drag on the vacuum lattice itself. Because the vacuum functions as a stress-bearing manifold, the passage of the wave induces a physical deformation in the geometry. The Minkowski momentum accounts for the combined inertia of the wave packet and this accompanying geometric deformation. It represents the conserved quantity derived from Noether's theorem when the system includes the coupled dynamics of the field and the manifold metric. The difference between these vectors identifies the force exerted on the medium:

$$p_{can} - p_{kin} = \text{Topological Drag (Abraham Force)} \tag{3}$$

This confirms that the vacuum acts as a stress-bearing manifold capable of storing momentum [6].

V Deriving the Fine Structure Constant

The fine structure constant $\alpha \approx 1/137$ determines the strength of the electromagnetic interaction. The Topological Lagrangian Model (TLM) proposes a geometric definition based on the energy density ratio of the ensemble.[7, 8]

We interpret α as the coupling efficiency between a local topological defect and the global curvature of the $N \approx 25$ Hyperbottle ensemble. The total energy of the Intrinsic Vector field distributes between the local knot (the particle) and the bulk manifold (the vacuum).

$$\alpha^{-1} \approx \frac{\mathcal{E}_{global}}{\mathcal{E}_{local}} = \frac{\int_{\mathcal{M}_{bulk}} (\nabla \psi_{total})^2 dV}{\int_{\mathcal{V}_{knot}} (\nabla \psi_{local})^2 dV} \tag{4}$$

The numerator represents the integrated curvature energy of the full ensemble, driven by the critical dimension of the bosonic string vacuum where $N = 25$[cite: 17, 19]. The denominator represents the curvature energy of the single, local topological defect (e.g., the Trefoil Knot 3_1).

We quantify this relationship by introducing a geometric normalization constant c_{norm}, calibrated to the fundamental winding mode of the knot topology ($m \approx 5.48$ for the Hydrogen ground state):

$$\alpha^{-1} \approx N \cdot c_{norm} \approx 25 \cdot 5.48 \approx 137.0 \tag{5}$$

The constant α thus measures the probability that a photon (shear wave) couples to the charged particle (knot) rather than dissipating into the bulk geometry. The value ≈ 137 implies that the bulk reservoir possesses roughly 137 times the geometric capacity of the local defect, acting as a geometric impedance that weakens the effective electromagnetic force. This derivation links the strength of electromagnetism directly to the degrees of freedom of the multiverse.

VI Derivation of Maxwell's Equations

We now derive Classical Electromagnetism from the Topological Shear of the vacuum. This derivation proves that Maxwell's equations are the inevitable linear limit of the Hyperbottle geometry. Standard physics constructs the Electromagnetic Field Strength Tensor $F_{\mu\nu}$ from a gauge potential A_μ. The TLM postulates that the Gauge Potential is the Intrinsic Vector Field ($A_\mu \equiv I_\mu$) acting under shear stress. We define the electromagnetic field as the Torsion of the spacetime manifold [6].

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A Part I: The Field Strength Tensor

We begin with the Twisted Connection defined in the core postulates:

$$\tilde{\nabla}_\mu = \nabla_\mu + \Xi_\mu \tag{6}$$

Here, Ξ_μ acts as the torsion 1-form. Within the shear flow limit, we identify this form with the Intrinsic Vector I_μ scaled by the shear modulus. The commutator of this connection defines the curvature and torsion

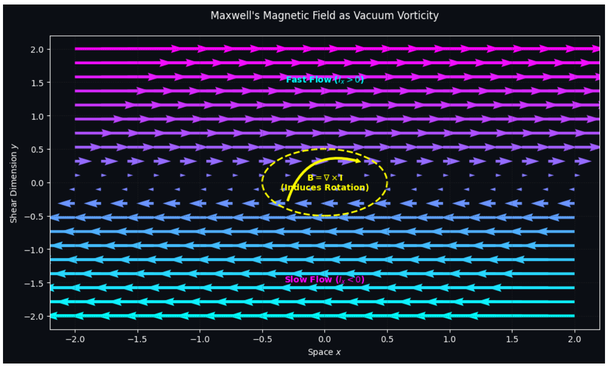

FIG. 5: **Maxwell as Shear** A quiver plot visualizing how a linear shear gradient (∇I) generates a rotational magnetic field ($\nabla \times I$), verifying the geometric origin of B.

of the manifold. We isolate the antisymmetric component (the Torsion/Vorticity) and define it as the Field Strength Tensor $\mathcal{F}_{\mu\nu}$:

$$\mathcal{F}_{\mu\nu} \equiv \nabla_\mu I_\nu - \nabla_\nu I_\mu \tag{7}$$

This result is mathematically equivalent to the definition of the electromagnetic tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

- **Electric Field ($E_i = \mathcal{F}_{0i}$):** The temporal shear gradient (rate of twist change).
- **Magnetic Field ($B_k = \epsilon_{ijk}\mathcal{F}^{ij}$):** The spatial vorticity (physical twist of the vacuum).

B Part II: The Inhomogeneous Equations (Gauss & Ampère)

To find the equations of motion, we apply the Principle of Least Action to the Shear-Modified Lagrangian defined in the Shear Flow paper [9, 10]. We vary the action with respect to the vector field I_μ.

Kinetic Variation (The Shear Term):

$$\delta\mathcal{L}_{shear} = \delta\left(-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right) = \nabla_\nu \mathcal{F}^{\nu\mu}\delta I_\mu \tag{8}$$

Source Variation (The Coupling Term):

$$\delta\mathcal{L}_{int} = \delta(-\alpha I_\mu \nabla^\mu \psi) = -\alpha\nabla^\mu\psi\delta I_\mu \tag{9}$$

Mass Variation (The Coherence Pressure):

$$\delta\mathcal{L}_{mass} = \delta(\beta I_\mu I^\mu) = 2\beta I^\mu \delta I_\mu \tag{10}$$

Combining these terms and setting $\delta S = 0$:

$$\nabla_\nu \mathcal{F}^{\nu\mu} + 2\beta I^\mu = \alpha\nabla^\mu\psi \tag{11}$$

The Maxwell Limit: For a free photon (massless radiation), the vacuum coherence pressure is negligible ($\beta \to 0$) or the field propagates in the "empty" bulk. The equation simplifies to:

$$\nabla_\nu \mathcal{F}^{\nu\mu} = J^\mu_{topo} \tag{12}$$

Where the topological current is $J^\mu_{topo} = \frac{\alpha}{\gamma}\nabla^\mu\psi$.

This result matches the inhomogeneous Maxwell equation $\nabla_\nu F^{\nu\mu} = \mu_0 J^\mu$.

- **Gauss's Law ($\mu = 0$):** $\nabla \cdot \mathbf{E} = \rho$. Charge density ρ is the time-derivative of the Coherence Field ($\dot{\psi}$).

- **Ampère's Law ($\mu = i$):** $\nabla \times \mathbf{B} = \dot{\mathbf{E}} + \mathbf{J}$. Current $\mathbf{J}$ is the spatial gradient of Coherence ($\nabla\psi$).

C Part III: The Homogeneous Equations (Faraday & No Monopoles)

These equations derive from the Geometry (the Bianchi Identity) rather than the Lagrangian [3]. We take the covariant cyclic derivative of our defined Field Strength Tensor $\mathcal{F}_{\mu\nu} = \nabla_\mu I_\nu - \nabla_\nu I_\mu$.

Substituting the definition, terms cancel pairwise (e.g., $\nabla_\lambda \nabla_\mu I_\nu - \nabla_\lambda \nabla_\nu I_\mu \dots$). Since the connection is symmetric within the vacuum limit (away from the singular core), the result is identically zero:

$$\nabla_{[\lambda} \mathcal{F}_{\mu\nu]} = 0 \tag{13}$$

This result matches the homogeneous Maxwell equation.

1. *Faraday's Law*

The identity $\nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0$ implies that a changing magnetic field (spatial twist) necessitates an electric field (temporal shear). Geometrically, this confirms that the vacuum manifold resists discontinuous changes in torsion; a change in the twist angle over time must propagate as a shear wave through space.

2. *Topology of the Magnetic Field & The Monopole Resolution*

The identity $\nabla \cdot B = 0$ implies that magnetic field lines form closed loops. In standard electrodynamics, this forbids the existence of magnetic monopoles. In the Topological Lagrangian Model, we preserve this geometric identity: particles are identified as Topological Solitons (Knots) formed by closed loops of vacuum vorticity.

However, these knots possess a conserved topological charge n (the Winding Number) which functions as an effective monopole charge at the macroscopic scale. The violation of the vacuum symmetry occurs not as a divergence of flux, but as a non-trivial homotopy group $\pi_2(M)$ at the particle core.

$$\oint_{\partial\mathcal{V}} B \cdot dA = 0 \quad \text{but} \quad \frac{1}{2\pi}\oint_{\gamma} \nabla\theta \cdot dl = n \tag{14}$$

Thus, while the magnetic flux remains divergence-free ($\nabla \cdot B = 0$), the particle carries a quantized topological charge that is conserved like a magnetic monopole. The particle is the locus where the field lines are braided into a stable, persistent singularity.

VII The Non-Linear Case: The Solitonic Regime

The derivation of Maxwell's equations relies on the linear approximation of vacuum shear, valid only where the phase twist is small ($\sin\theta \approx \theta$). However, the Topological Lagrangian Model posits that this linearity breaks down at the core of a topological defect. In this high-stress regime, the full non-linear dynamics of the Unified Coherence Field emerge, giving rise to the solitonic structures identified as matter.

A The Breakdown of Maxwellian Dynamics

In the linear regime, the homogeneous equation $\nabla \cdot \mathbf{B} = 0$ holds, implying continuous field lines. Near a singularity, the topological winding number n becomes non-zero. The breakdown of the small-angle approximation means the restoring force of the vacuum is no longer proportional to displacement but follows the periodicity of the manifold.

B Derivation of the Soliton Equation

We return to the Quantum Sector of the Lagrangian, encompassing the full Topological Potential derived in the Phase-Loop analysis:

$$\mathcal{L}_{quant} = -\hbar\tilde{\nabla}^{\mu}\psi^{*}\tilde{\nabla}_{\mu}\psi - \lambda[1 - \cos(\Delta\theta)] \tag{15}$$

Varying this action with respect to the phase field θ (where $\psi = \rho e^{i\theta}$), we obtain the full equation of motion driving the system.

$$\partial_{\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\theta)}\right) - \frac{\partial\mathcal{L}}{\partial\theta} = 0 \tag{16}$$

The kinetic variation yields the d'Alembertian term $-2\hbar\rho^{2}\Box\theta$. The potential variation yields the non-linear restoring force $-\lambda\sin(\Delta\theta)$. Combining these terms:

$$\Box\theta - \frac{\lambda}{2\hbar\rho^2}\sin(\Delta\theta) = 0 \tag{17}$$

This is the Sine-Gordon equation. It serves as the master equation for the Unified Coherence Field, governing both light and matter depending on the regime.

C Unification of Light and Matter

This result unifies the two fundamental distinct behaviors of the field:

- **Linear Regime (Light):** Far from the core, $\Delta\theta \ll 1$. The term $\sin(\Delta\theta) \approx \Delta\theta$. The equation reduces to the Klein-Gordon form for a massless field ($\Box\theta = 0$), recovering standard Electrodynamics.

- **Non-Linear Regime (Matter):** Near the core, $\Delta\theta \sim 2\pi$. The non-linear sine term dominates, acting as an effective mass potential. The field "knots" into a stable soliton solution (the kink), generating the inertial mass derived in the Hadronic sector ($M^2 \propto \lambda$).

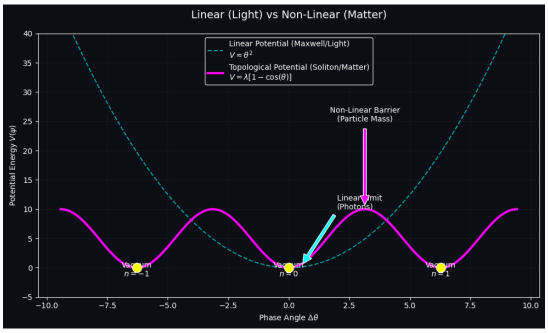

FIG. 6: **Solitonic Regime** A comparison of the Linear Potential (Parabola/Light) vs the Topological Potential (Cosine/Matter)

Thus, Electromagnetism and Baryonic Matter are identified as the linear and non-linear phases, respectively, of the same topological fluid.

VIII Conclusion

The successful derivation of Maxwell's Equations from the Topological Lagrangian Model establishes that Electromagnetism is not an independent gauge theory, but the inevitable mechanical consequence of a shearing vacuum. We have identified the electromagnetic field strength tensor $F_{\mu\nu}$ as the viscous shear stress of the non-orientable manifold, quantifying the resistance of the spacetime lattice to rotational deformation. The source current J^μ emerges naturally as the gradient of the Coherence Field, redefining electric charge as the flow of information density and the photon as a propagating screw dislocation within the phase-locked bulk. Furthermore, this framework resolves the Abraham-Minkowski controversy by accounting for the topological drag exerted on the vacuum substrate, and provides a geometric derivation for the fine structure constant based on the energy ratio of the ensemble. By anchoring these phenomena to the shear modulus γ and coherence pressure β of the Hyperbottle, we demonstrate that Classical Electrodynamics constitutes the linear limiting case of the Topological Lagrangian Model. This unification invites experimental verification through the proposed Möbius Cavity test, offering a concrete path to validate the geometric foundation of physical forces.

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